

Sound modes in hot nuclear matter

V.M.Kolomietz^{1,2)} and S.Shlomo¹⁾

¹⁾ *Cyclotron Institute, Texas A&M University, College Station, TX 77843, USA*

²⁾ *Institute for Nuclear Research, 03028 Kiev, Ukraine*

Abstract

The propagation of the isoscalar and isovector sound modes in a hot nuclear matter is considered. The approach is based on the collisional kinetic theory and takes into account the temperature and memory effects. It is shown that the sound velocity and the attenuation coefficient are significantly influenced by the Fermi surface distortion (FSD). The corresponding influence is much stronger for the isoscalar mode than for the isovector one. The memory effects cause a non-monotonous behavior of the attenuation coefficient as a function of the relaxation time leading to a zero-to-first sound transition with increasing temperature. The mixing of both the isoscalar and the isovector sound modes in an asymmetric nuclear matter is evaluated. The condition for the bulk instability and the instability growth rate in the presence of the memory effects is studied. It is shown that both the FSD and the relaxation processes lead to a shift of the maximum of the instability growth rate to the longer wave length region.

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I. INTRODUCTION

In the vicinity of the equilibrium state nuclear matter is stable with respect to particle density and Fermi surface distortions and the excitation of both the isoscalar and the isovector sound modes is possible. Propagation of an isoscalar sound wave in nuclear matter depends crucially on the Landau parameter F_0 in the quasiparticle interaction amplitude and on the relaxation processes. For zero temperature, there is an underdamped zero-sound mode at $F_0 > 0$ because the phase velocity of the sound wave exceeds the speed of particles inside the Fermi sphere. A strong Landau damping appears at $-1 < F_0 < 0$ where a nonzero transfer of energy from the wave to particles is possible [1] and the wave transforms to an overdamped mode. The picture of propagation of the zero-sound wave is essentially more complicated in the case of a hot nuclear matter. Due to the existence of the temperature tail of the equilibrium distribution function, the phase conditions for the Landau damping are fulfilled here for positive values of the F_0 and a possibility for a propagation of the sound wave appears in the region $-1 < F_0 < 0$ [2].

The zero sound is transformed to the first-sound mode in the limit of strong interaction, $|F_0| \gg 1$, [3] or in the frequent collision regime at high temperatures [4]. It is necessary to stress that the sound velocity c is directly related to the nuclear matter incompressibility coefficient K for the first-sound mode only. In this case, one has $c \approx c_1 = \sqrt{K/9m}$, where c_1 is the velocity of the first sound. In general, the sound velocity c is a complicated function of both K and the dimensionless collisional parameter $\omega\tau$, where τ is the relaxation time and ω is the eigenfrequency of the sound mode. In the present work we obtain a simple analytical expression for the sound velocity c which provides a description for both the frequent- and rare-collision limit as well as for the intermediate cases. Special attention is paid to the propagation of the isovector sound and for the zero- to first- sound transition for this mode. It is well known [5] that hydrodynamic approaches like the Goldhaber-Teller or Steinwedel-Jensen models give a reasonable description of the nuclear isovector giant resonances (IVGR). However it is not the case for the isoscalar giant resonances (ISGR), where the Fermi surface distortion effects plays an important role [6–9] and the traditional hydrodynamic model can not be applied. In our approach, this situation is directly related to the peculiarities of the propagation of both isovector and isoscalar sounds.

The interparticle collisions on the distorted Fermi surface lead to the collisional damping of the sound wave. Two limiting regimes $\omega\tau \rightarrow 0$ and $\omega\tau \rightarrow \infty$ provide the existence of the non-damped first and zero sounds, respectively, [1]. In what follows we will use the collisional kinetic theory, taking into account the memory effects on the collision integral [1,10–12]. We will discuss a special feature of the temperature dependence of the attenuation of the sound mode in hot nuclear matter. In particular, we will show that memory effects lead to a bell-shaped form of the attenuation coefficient κ as a function of the temperature, providing a correct behavior of κ in both limiting regimes $\omega\tau \rightarrow 0$ and $\omega\tau \rightarrow \infty$.

With decreasing bulk density or increasing temperature the nuclear matter reaches the regions of mechanical or thermodynamical instabilities with respect to small particle density distortions [3,13–17] and to separation into liquid and gas phases [18,19]. The general instability condition of the Fermi liquid reads $1 + F_k/(2k + 1) < 0$ [20], where F_k is the Landau's parameter in the expansion of the quasiparticle interaction amplitude in Legendre polynomial [1]. However the development of instability depends not only on the equation of

state, but also on the dynamical effects such as the dynamical Fermi-surface distortion (FSD) effect [21]. The FSD effects strongly reduce, by the factor $\sim (|F_0| - 1)^{1/2}$, the instability growth rate Γ in the unstable region $0 < -1 - F_0 \ll 1$ [3]. In the present work, we will consider the influence of the Fermi-surface distortion, relaxation and memory effects on the instability growth rate Γ .

The plan of the paper is as follows. In Sec. II we derive the general equation of motion for the particle density vibrations in the presence of the memory effects. We introduce the renormalized incompressibility coefficient and the viscosity coefficient which are frequency dependent due to the memory effect. In Sec. III we obtain a simple expression for the refraction and attenuation coefficients for both the isoscalar and the isovector sound waves which propagate in a hot nuclear matter. We consider also the mixing of the isoscalar and isovector modes in an asymmetric nuclear matter taking into account the relaxation and the memory effects. The development of the bulk instability in a low density nuclear matter in the presence of the relaxation and the memory effects is considered in Sec. IV. The conclusion is given in Sec. V.

II. SOUND PROPAGATION IN HOT NUCLEAR MATTER

We start from the collisional kinetic equation [1]

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m} \frac{\partial f}{\partial \mathbf{r}} - \frac{\partial V}{\partial \mathbf{r}} \frac{\partial f}{\partial \mathbf{p}} = \delta \text{St}(t). \quad (1)$$

Here $\delta \text{St}(t)$ is the collisional integral, $V \equiv V(\mathbf{r}, t)$ is the self-consistent mean field and $f \equiv f(\mathbf{r}, \mathbf{p}, t)$ is the Wigner distribution function in which we will take into account only the distortion of the Fermi sphere with multipolarities $\ell \leq 2$

$$f = f_s + \delta f, \quad \delta f = \sum_{\ell=1}^2 \delta f_\ell. \quad (2)$$

The distribution $f_s \equiv f_s(\mathbf{r}, \mathbf{p}, t)$ corresponds to the spherical Fermi surface and δf represents both the quadrupole deformation and the displacement of the Fermi surface. For small deviations from a Fermi sphere the right-hand side (RHS) $\delta \text{St}(t)$ of (1) is a collision integral linearized in δf and it can be represented in the form

$$\delta \text{St}(t) = \int_{-\infty}^t dt' A(t - t') \delta f(t'), \quad (3)$$

which takes into account the memory effects due to the memory kernel $A(t - t')$. In this paper we will not use an explicit form of A . Below we will need $\delta \text{St}(t)$ only for periodic oscillation of δf with the eigenfrequency ω . Assuming the restriction $\ell \leq 2$ for the Fermi surface distortion, the collision integral (3) can be written in the form of the extended τ -approximation, see Refs. [22,23],

$$\delta \text{St}(t) = -\frac{\delta f_2}{\tau_{r,\omega}}, \quad (4)$$

where the relaxation time $\tau_{r,\omega}$ is ω -dependent due to the memory effects.

The assumption in Eq. (2) allows us to reduce the collisional kinetic equation (1) to the local equations of motion for the particle density $\rho \equiv \rho(\mathbf{r}, t)$, the displacement field $\chi_\alpha \equiv \chi_\alpha(\mathbf{r}, t)$ and the pressure tensor $P_{\alpha\beta} \equiv P_{\alpha\beta}(\mathbf{r}, t)$ derived as the lowest \mathbf{p} -moments of the distribution function $f(\mathbf{r}, \mathbf{p}, t)$:

$$\rho = \int \frac{g d\mathbf{p}}{(2\pi\hbar)^3} f, \quad \frac{\partial \chi_\alpha}{\partial t} = \frac{1}{\rho} \int \frac{g d\mathbf{p}}{(2\pi\hbar)^3} \frac{p_\alpha}{m} f, \quad P_{\alpha\beta} = \frac{1}{m} \int \frac{g d\mathbf{p}}{(2\pi\hbar)^3} (p_\alpha - mu_\alpha)(p_\beta - mu_\beta) f. \quad (5)$$

Here $u_\alpha = \partial \chi_\alpha / \partial t$ is the velocity field and g is the spin-isospin degeneracy factor. Taking the first two \mathbf{p} -moments of Eq. (1) one obtains, see Refs. [22–24],

$$m\rho \frac{\partial^2}{\partial t^2} \chi_\nu + \left(\frac{\partial}{\partial r_\nu} P + \rho \frac{\partial}{\partial r_\nu} V \right) + \frac{\partial}{\partial r_\mu} P'_{\nu\mu} = 0, \quad (6)$$

where P is the pressure due to motion of nucleons without distortion of the Fermi sphere and $P'_{\nu\mu}$ is associated with quadrupole distortion of the Fermi surface: $P_{\alpha\beta} = P\delta_{\alpha\beta} + P'_{\alpha\beta}$. The pressure tensor $P'_{\nu\mu}$ is responsible for the dissipative processes. Taking the second \mathbf{p} -moment of Eq. (1) one obtains the following equation for the pressure tensor

$$\frac{\partial}{\partial t} P_{\alpha\beta} + \frac{\partial}{\partial r_\nu} u_\nu P_{\alpha\beta} + P_{\nu\beta} \frac{\partial}{\partial r_\nu} u_\alpha + P_{\nu\alpha} \frac{\partial}{\partial r_\nu} u_\beta = -P'_{\alpha\beta} / \tau_{r,\omega}. \quad (7)$$

The equations of motion (6) and (7) are closed. They can be applied to both the isoscalar and isovector sound excitations. Let us consider the isoscalar compression mode. To simplify the problem, we can rewrite the expression in the parenthesis in Eq. (6) near the equilibrium value of the density ρ_{eq} as

$$\frac{\partial}{\partial r_\nu} P + \rho \frac{\partial}{\partial r_\nu} V = \rho \frac{\partial}{\partial r_\nu} \frac{\delta \epsilon}{\delta \rho} \approx \rho_{\text{eq}} \frac{\partial}{\partial r_\nu} \left[\left(\frac{\delta^2 \epsilon}{\delta \rho^2} \right)_{\text{eq}} \delta \rho \right], \quad (8)$$

where index "eq" refers the equilibrium state, ϵ is the energy density of particles

$$\epsilon = \epsilon_{\text{kin}} + \epsilon_{\text{pot}}, \quad (9)$$

ϵ_{kin} is the kinetic energy density

$$\epsilon_{\text{kin}} = \frac{3}{2} P = \int \frac{g d\mathbf{p}}{(2\pi\hbar)^3} \frac{p^2}{2m} f_s = \frac{3}{10} \frac{\hbar^2}{m} \left(\frac{3\pi^2}{g} \right)^{2/3} \rho^{5/3} \quad (10)$$

and ϵ_{pot} is the potential energy density which is related to the mean field V by

$$V = \delta \epsilon_{\text{pot}} / \delta \rho. \quad (11)$$

Note that $(\delta \epsilon / \delta \rho)_{\text{eq}}$ is the chemical potential, which does not depend on the space coordinate \mathbf{r} , for the equilibrium state of the nucleus. We have used this fact when deducing Eq. (8). Solving Eq. (7) with respect to $P'_{\alpha\beta}$, using Eq. (8) and the continuity equation $\delta \rho = \rho - \rho_{\text{eq}} = -\text{div}(\rho_{\text{eq}} \chi)$, we obtain an equation for the density vibration in the form

$$\omega^2 \delta \rho + (K'_\omega/9m) \nabla^2 \delta \rho = i\omega(4\eta_\omega/3m\rho_{\text{eq}}) \nabla^2 \delta \rho. \quad (12)$$

Here

$$K'_\omega = K + 8(\epsilon_{\text{kin}}/\rho)_{\text{eq}} \text{Im} \left(\frac{\omega \tau_{r,\omega}}{1 - i\omega \tau_{r,\omega}} \right), \quad (13)$$

where $K \equiv 9(\delta^2 \epsilon / \delta \rho^2)_{\text{eq}} \rho_{\text{eq}}$ is the static incompressibility and η_ω is the viscosity coefficient

$$\eta_\omega = \text{Re} \left(\frac{\tau_{r,\omega}}{1 - i\omega \tau_{r,\omega}} \right) P_{\text{eq}}. \quad (14)$$

We point out that there is a significant difference between the static nuclear incompressibility coefficient, K , i.e., derived as a stiffness coefficient with respect to a change in the bulk density, and the dynamic one, K'_ω of Eq. (13), associated with the sound propagation. This difference is due to the second term on the RHS of Eq. (13) caused by the Fermi-surface distortion effects. The quantity η_ω in Eq. (14) determines the time irreversible contribution to the pressure tensor $P'_{\alpha\beta}$ and can be considered as the viscosity coefficient due to the relaxation occurring on the distorted Fermi surface. Expression (14) is valid independently of the nucleon's collision rate. The viscosity goes to zero in both the rare, $\tau_{r,\omega} \rightarrow \infty$, and frequent, $\tau_{r,\omega} \rightarrow 0$, collision limits.

III. DISPERSION RELATION, MEMORY EFFECTS AND DAMPING

Assuming a plane wave solution $\delta \rho \sim \exp(i\mathbf{q} \cdot \mathbf{r} - i\omega t)$ one obtains from Eq. (12) the following dispersion relation

$$\omega^2 = (K'_\omega/9m)q^2 - i\omega(4\eta_\omega/3m\rho_{\text{eq}})q^2. \quad (15)$$

The solution of this equation defines the complex wave number q (ω is real). A simple solution to Eq. (15) can be obtained in two limiting cases of the frequent collision (first sound) regime, $\omega \tau_{r,\omega} \rightarrow 0$, and the rare collision (zero sound) regime, $\omega \tau_{r,\omega} \rightarrow \infty$. The sound velocity $c = \omega/q$ is given by

$$c = c_1 = \sqrt{K/9m} \text{ if } \omega \tau_{r,\omega} \rightarrow 0 \text{ and } c = c_0 = \sqrt{(K + \Delta K)/9m} \text{ if } \omega \tau_{r,\omega} \rightarrow \infty, \quad (16)$$

where $\Delta K \approx 8(\epsilon_{\text{kin}}/\rho)_{\text{eq}} \approx (24/5) e_F \approx 200$ MeV (we adopted the kinetic Fermi energy $e_F \approx 40$ MeV). We point out that the value of ΔK is comparable with the static incompressibility $K \approx 220$ MeV and we have $c_0 \approx \sqrt{2}c_1$. The factor $\sqrt{2}$ in this relation is due to the restriction $\ell \leq 2$ for the multipolarity ℓ of the Fermi surface distortion. In a general case of arbitrary ℓ this factor is increased to $\sqrt{3}$ [1]. The result (16) means that in contrast to the first sound (frequent collision) regime, the sound velocity of the compression mode can not, in general, be used directly to extract the static incompressibility of K because of the additional contribution from the Fermi surface distortion effects which result in the renormalization of the incompressibility K into K'_ω .

Using both asymptotic sound velocity c_1 and c_0 , the solution to the dispersion relation (15) can be written as

$$q = \frac{\omega}{c_0}(n + i\kappa), \quad (17)$$

where the refraction coefficient n and the attenuation coefficient κ (both real) are obtained from the following equation

$$n + i\kappa = \sqrt{\frac{1 - i\omega\tau_{r,\omega}}{(c_1/c_0)^2 - i\omega\tau_{r,\omega}}}. \quad (18)$$

In the frequent collision (first sound) regime we obtain from Eq. (18)

$$n = \frac{c_0}{c_1}, \quad \kappa = \omega\tau_{r,\omega}(c_0/2c_1)[(c_0/c_1)^2 - 1] \quad \text{if} \quad \omega\tau_{r,\omega} \ll 1. \quad (19)$$

In the opposite case of the rare collision (zero sound) regime we obtain

$$n = 1, \quad \kappa = [1 - (c_1/c_0)^2]/(2\omega\tau_{r,\omega}) \quad \text{if} \quad \omega\tau_{r,\omega} \gg 1. \quad (20)$$

The attenuation coefficient κ in both limiting regimes is a complicated function of the frequency ω because of the memory effect in the relaxation time $\tau_{r,\omega}$. In the case of sound propagation in hot nuclear matter, the competition between the temperature smoothing effects in the equilibrium distribution function and dynamical distortions of the particle momentum distribution leads to the following expression for the relaxation time [1,11,12]

$$\tau_{r,\omega} = \frac{\tau_0}{T^2 + \xi(\hbar\omega)^2}, \quad (21)$$

where T is the temperature of nuclear matter and the ω -dependence of $\tau_{r,\omega}$ is due to the memory effects in the collision integral. Below we will use $\xi = 1/4\pi^2$ [1] and $\tau_0 = \alpha\hbar$, $\alpha = 9.2$ MeV (for the isoscalar mode) [26].

Equations (15) and (18) are valid for arbitrary collision times $\tau_{r,\omega}$ and thus describe both the zero and the first sound limit as well as the intermediate cases. From it one can obtain the leading order terms in the different limits mentioned. In Fig. 1 we have plotted both coefficients n and κ as obtained from Eq. (18). In the high temperature limit, the system goes to the frequent collision (first sound) regime with the saturated refraction coefficient $n \approx c_0/c_1 \approx \sqrt{3}$ (we use the factor $\sqrt{3}$ instead of $\sqrt{2}$ assuming the contribution of the higher multipoles $\ell > 2$ in the Fermi surface distortion as was mentioned above) and the attenuation coefficient $\kappa \sim \tau_{r,\omega} \sim 1/T^2$. In the opposite low temperature limit, the system is close to the zero sound regime with $n \approx 1$. We point out a shift of both n and κ by nonzero values at $T \rightarrow 0$. This is due to the memory effect in the relaxation time $\tau_{r,\omega}$ of Eq. (21): in the very high frequency limit, the system can exist close to the first sound regime at $n \approx \sqrt{3}$ even at zero temperature. The position of the maximum of $\kappa(T)$ in Fig. 1 can be interpreted as the transition temperature T_{tr} of zero- to first- sound regimes in a hot Fermi system. The value of T_{tr} depends slightly on the sound frequency ω and it is shifted to smaller values with the increase of ω .

Let us consider now the isovector sound mode in a symmetric nuclear matter with $\rho_{n,\text{eq}} = \rho_{p,\text{eq}}$, where $\rho_{n,\text{eq}}$ and $\rho_{p,\text{eq}}$ are the equilibrium neutron and proton density respectively. The general equations of motion (6) and (7) are still correct. However the energy density ϵ in Eq.

(8) is related now to the symmetry energy E_{symm} . The corresponding first sound velocity c_1 for the isovector mode is given by, [27] Ch. 6,

$$c_1 = \sqrt{2E_{\text{symm}}/m}, \quad (22)$$

where $E_{\text{symm}} = (1/3)e_F(1 + F'_0) \approx 30$ MeV, and F'_0 is the isovector Landau parameter in the quasiparticle interaction amplitude. The zero sound velocity c_0 for the isovector mode can be found from Eqs. (6) and (7) in the rare collision limit $\tau_{r,\omega} \rightarrow \infty$. Taking into account Eq. (11), one obtains, see also Eq. (16),

$$c_0 = \sqrt{2(E_{\text{symm}} + \Delta E_{\text{symm}})/m}, \quad (23)$$

where $\Delta E_{\text{symm}} \approx (4/9)(\epsilon_{kin}/\rho)_{\text{eq}} \approx (4/15) e_F \approx 10$ MeV. We point out that, in contrast to the isoscalar mode, the Fermi surface distortion effect leads to a relatively small increase of the isovector zero sound velocity c_0 , Eq. (23), with respect to the first sound one c_1 , Eq. (22). The dispersion relation (15) takes the form

$$\omega^2 = (2E'_{\text{symm},\omega}/m)q^2 - i\omega(4\eta_\omega/3m\rho_{\text{eq}})q^2, \quad (24)$$

where

$$E'_{\text{symm},\omega} = E_{\text{symm}} + (4/9)(\epsilon_{kin}/\rho)_{\text{eq}} \text{Im} \left(\frac{\omega\tau_{r,\omega}}{1 - i\omega\tau_{r,\omega}} \right). \quad (25)$$

All relations (17)-(21) are still correct in the isovector case if both asymptotic velocity c_0 and c_1 are taken from Eqs. (22) and (23). In Fig. 2 we have plotted the coefficients n and κ as obtained from Eq. (18) for the isovector mode with the collision parameter $\alpha = 4.6$ MeV [26]. We point out that the transition temperature T_{tr} of zero- to first- sound regimes for the isovector mode is significantly smaller than T_{tr} for the isoscalar one.

In an asymmetric nuclear matter, both the isovector and the isoscalar modes are dependent on each other. The particle density fluctuation $\delta\rho$ takes a bispinor form $\delta\rho = (\delta\rho_+, \delta\rho_-)$, where $\delta\rho_+$ and $\delta\rho_-$ are the isoscalar and isovector components respectively. A solution of the corresponding equations of motion (6) and (7) leads to the following dispersion relation, see also Eqs. (15) and (24),

$$\text{Det} \begin{pmatrix} \omega^2 - (K'_\omega/9m)q^2 + i\omega(4\eta_\omega/3m\rho_{\text{eq}})q^2 & (Ix/m)q^2 + \mathcal{O}(I^2) \\ (Iy/m)q^2 + \mathcal{O}(I^2) & \omega^2 - (E'_{\text{symm},\omega}/9m)q^2 + i\omega(4\eta_\omega/3m\rho_{\text{eq}})q^2 \end{pmatrix} = 0. \quad (26)$$

Here, $I = (\rho_n - \rho_p)_{\text{eq}}/(\rho_n + \rho_p)_{\text{eq}} \ll 1$ is the asymmetry parameter, ρ_n and ρ_p are the neutron and proton densities respectively, $\rho_{\text{eq}} = (\rho_n + \rho_p)_{\text{eq}}$ and the coupling constants x and y are given by

$$x = -(2/9)(4\epsilon_F - K/3), \quad y = -(2/9)(4\epsilon_F + K/6 - 9E_{\text{symm}}). \quad (27)$$

As is seen from Eq. (27), the eigenfrequency ω and the corresponding sound velocity for both the isoscalar and the isovector modes are independent of each other in the linear order of the asymmetry parameter I . The structure of bispinor $\delta\rho = (\delta\rho_+, \delta\rho_-)$ is different for

both the isoscalar-like and the isovector-like modes. For the isoscalar-like mode with the eigenfrequency ω_{is} given by a solution to Eq. (15), the main contribution to the bispinor $\delta\rho$ is due to the isoscalar component $\delta\rho_+ \sim 1$ while the isovector component $\delta\rho_-$ is proportional to the asymmetry parameter I . Namely,

$$\left(\frac{\delta\rho_-}{\delta\rho_+}\right)_{\text{is}} = \frac{yq^2/m}{(\omega^2 - (E'_{\text{symm},\omega}/9m)q^2 + i\omega(4\eta_\omega/3m\rho_{\text{eq}})q^2)_{\omega=\omega_{\text{is}}}} I. \quad (28)$$

The opposite situation takes place for the isovector-like mode with the eigenfrequency ω_{iv} given by a solution to Eq. (24). In this case, one has $\delta\rho_- \sim 1$ and $\delta\rho_+ \sim I$.

IV. BULK INSTABILITY

Let us consider now the bulk instability regime $K < 0$ and introduce an instability growth rate $\Gamma = -i\omega$ (Γ is real, $\Gamma > 0$), see Refs. [13,21]. The amplitude of the density fluctuations, $\delta\rho \sim \exp(i\mathbf{q} \cdot \mathbf{r} - i\omega t) \sim \exp(\Gamma t)$, grows exponentially if $\Gamma > 0$. To prevent an unphysical infinite growth of the short wave length fluctuations (see dotted line in Fig. 3), we will take into account the velocity dependent contribution to the effective interparticle interaction. Due to the corresponding change in the selfconsistent mean field V in Eq. (6), an additional anomalous term $\sim q^4$ appears in the dispersion relation (15) and the equation for the instability growth rate Γ takes the following form [21]

$$\Gamma^2 = |c_1|^2 q^2 - \zeta(\Gamma) q^2 - \kappa_s q^4. \quad (29)$$

Here $c_1 = i\sqrt{|K|/9m}$ and

$$\zeta(\Gamma) = \frac{4}{3m} \frac{\Gamma\tau_r}{1 + \Gamma\tau_r} e_F, \quad (30)$$

where τ_r is the relaxation time $\tau_r = \alpha\hbar/T^2$ [4]. The constant κ_s in the anomalous dispersion term in Eq. (30) depends on the model. In the case of the effective Skyrme forces, one has [21] $\kappa_s = \hbar^2/9m^2 + (9t_1 - 5t_2)\rho_0/32m$, where t_1 and t_2 are the parameters of the velocity dependent part of the Skyrme forces [28]. We point out that the instability regime with $K < 0$ can be reached at a low bulk density ρ_0 . The incompressibility K is given by

$$K = 6e_F(1 + F_0)(1 + F_1/3)^{-1}. \quad (31)$$

Here, the Landau parameters F_k are related to the parameters t_n of the effective Skyrme forces. Namely,

$$F_0 = \frac{9\rho_0}{8e_F} \left[t_0 + \frac{3}{2} t_3 \rho_0 \right] \frac{m^*}{m} + 3 \left(1 - \frac{m^*}{m} \right), \quad F_1 = 3 \left(\frac{m^*}{m} - 1 \right), \quad (32)$$

where $m/m^* = 1 + m\rho_0(3t_1 + 5t_2)/8\hbar^2$. For the commonly used set of parameters t_n , the instability regime with $F_0 < -1$ is reached at $\rho_0 \lesssim 0.5\rho_{\text{eq}}$.

In Fig. 3 we have plotted the instability growth rate Γ as obtained from Eq. (29). The calculation was performed for the Skyrme force SIII. For the relaxation time τ_r we used

$\alpha = 9.2$ MeV and the bulk density ρ_0 was $\rho_0 = 0.3 \rho_{\text{eq}}$, where ρ_{eq} is the saturated density $\rho_{\text{eq}} = 0.1453 \text{ fm}^{-3}$. We also show in Fig. 3 the result for the nonviscous nuclear matter neglecting the anomalous dispersion term $\sim q^4$ and the Fermi surface distortion effect (dotted line). The non-monotonous behavior of the instability growth rate as a function of the wave number q is due to the anomalous dispersion term in Eq. (29), induced by the velocity dependent terms in the interparticle interaction. The instability growth rate Γ reaches a maximum, Γ_{max} , at a certain $q = q_{\text{max}}$ and Γ goes to zero at $q = q_{\text{crit}}$. The existence of the critical wave number $q_{\text{crit}} = |c_1|^2/\kappa_s$ for an unstable mode is a feature of the system with the anomalous dispersion term [13]. The distortion of the Fermi surface leads to a decrease of the critical value q_{crit} , i.e., the nuclear matter becomes more stable due to the FSD effect. We can also see that the presence of viscosity and the FSD effect lead to a shift of the position q_{max} of the maximum of $\Gamma(q)$ to the left. Thus, the instability of the nuclear matter with respect to short-wave-length density fluctuations decreases due to the viscosity and the FSD effect and the most unstable mode is shifted to the region of the creation of larger clusters in the disintegration of nuclear matter. For a saturated nuclear liquid one has for the force parameters $t_0 < 0$, $t_3 > 0$ and $t_s > 0$. However both values q_{max} and q_{crit} have a non-monotonous behavior as a function of the bulk density ρ_0 because of the additional ρ_0 -dependence of the Fermi energy e_F in Eqs. (31) and (32). The particle density dependence of the values of q_{max} and q_{crit} is shown in Fig. 4. The instability growth rate $\Gamma(q)$ as well as the values of q_{max} and q_{crit} are only slightly sensitive to the change of temperature at $T \lesssim 10$ MeV, where the temperature dependence of the bulk density ρ_0 can be neglected. A more sophisticated consideration is necessary near the critical temperature $T_{\text{crit}} \approx 17$ MeV where the nuclear matter is unstable with respect to the liquid-gas phase transition.

V. CONCLUSION

Starting from the collisional kinetic equation (1), we have derived the dispersion relations (26) and (29) for both the stable and the unstable regime of the density fluctuations in a heated nuclear matter. The dispersion relations are influenced strongly by the FSD effect and the anomalous dispersion term. The presence of the Fermi surface distortion enhances the stiffness coefficient for a stable mode and reduces the instability growth rate for an unstable one. There is a significant difference between the static nuclear incompressibility coefficient, K , i.e., derived as a stiffness coefficient with respect to a change in the bulk density, and the dynamic one, K'_ω associated with the zero sound velocity, see Eq. (13). The FSD effect is responsible for the collisional relaxation of the collective modes in the Fermi liquid and for the non-Markovian character of the nuclear matter viscosity (memory effect in the viscosity η_ω , Eq. (14)). The memory effects in the viscosity play an essential role in the description of the temperature dependence of the refraction coefficient n , see Eqs. (18) and (21). We have noted also the bell-shaped form of the attenuation coefficient κ as a function of the temperature T , see Figs. 1 and 2. This peculiarity of $\kappa(T)$ provides a new criterion for the determination of the transition temperature T_{tr} between the zero-sound and first-sound regimes in hot nuclear matter.

Our consideration provides a good basis for understanding the difference in the development of the spinodal instability in nuclear matter taking into account both the FSD and

the viscosity effect. We have shown that the instability growth rate $\Gamma(q)$ in an unstable nuclear matter with velocity dependent effective interparticle interaction is a non-monotonous function of the wave number q because of the anomalous dispersion term. The anomalous dispersion term removes an unphysical infinite growth of the short wave length fluctuations. The non-monotonous behavior of the instability growth rate $\Gamma(q)$ is characterized by two wave numbers q_{max} and q_{crit} . We point out that both the FSD effect and the relaxation processes lead to a shift of q_{max} to the longer wave length region, providing an increase of the relative yield of heavier clusters.

The main results were obtained assuming a quadrupole distortion of the Fermi surface. We point out, however, that the key expression (14) for the viscosity coefficient can be rewritten identically in the following form

$$\eta_{\omega} = \frac{3}{4} m \rho_{\text{eq}} (c_0^2 - c_1^2) \text{Re} \left(\frac{\tau_{r,\omega}}{1 - i\omega\tau_{r,\omega}} \right), \quad (33)$$

where c_1 and c_0 are the first and zero sound velocity respectively. This expression can also be established from a general consideration, see Ref. [29] Ch.8, and can be used for arbitrary multipolarities of the Fermi surface distortion.

VI. ACKNOWLEDGEMENTS

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FIGURES

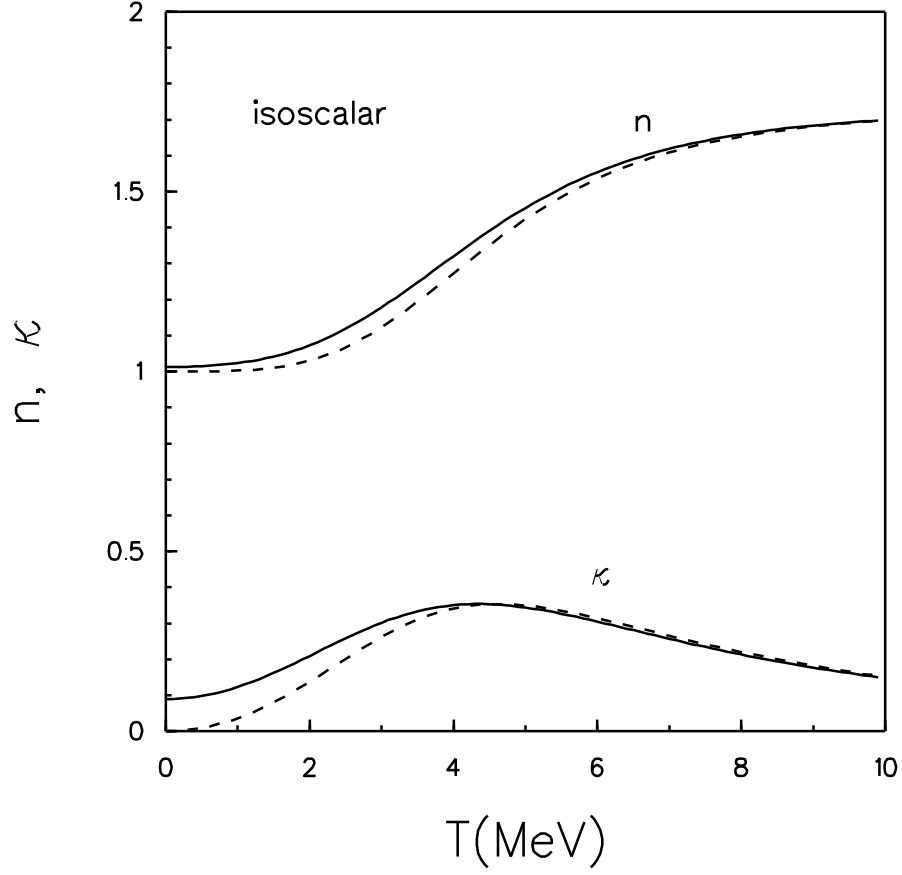


FIG. 1. Refraction, n , and attenuation, κ , coefficients of the isoscalar sound wave as functions of temperature. The calculation was performed for two eigenenergies $\hbar\omega = 1\text{MeV}$ (solid line) and $\hbar\omega = 1\text{eV}$ (dashed line).

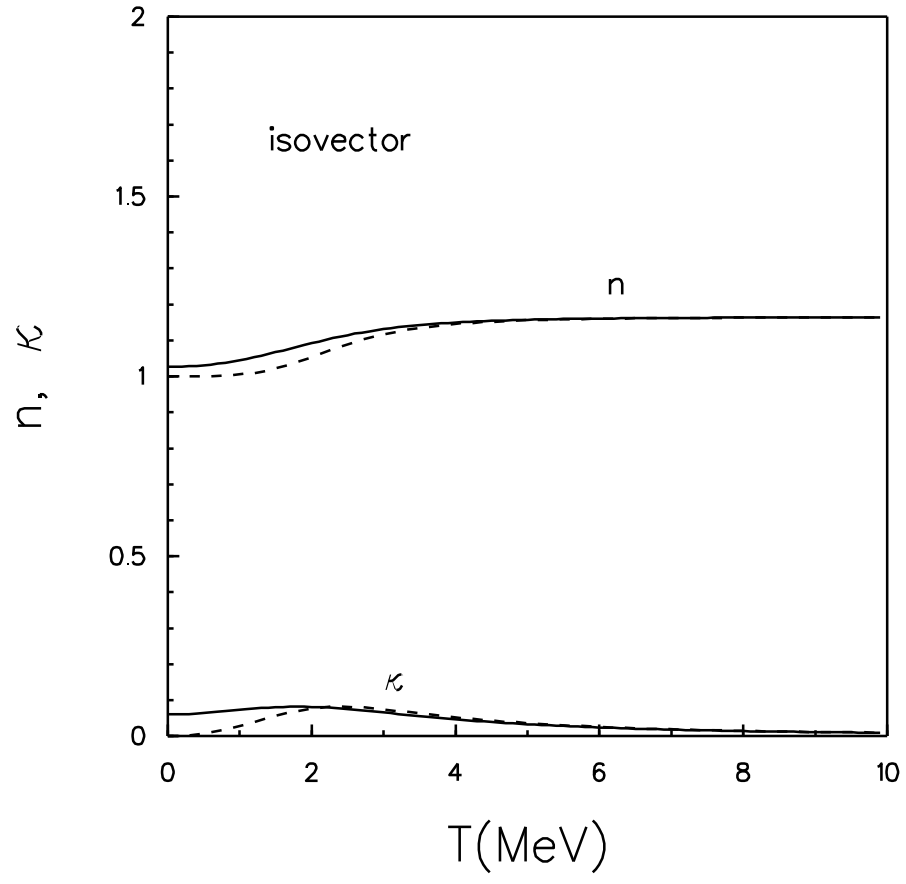


FIG. 2. Same as Fig. 1 for isovector mode.

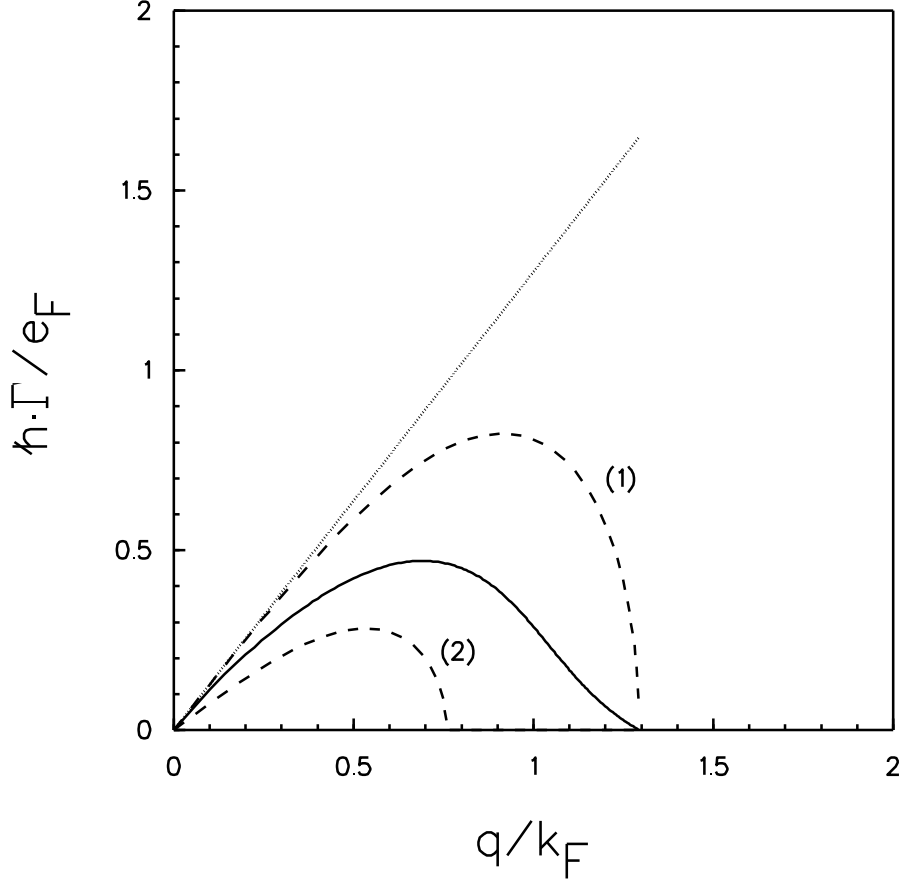


FIG. 3. Dependence of the instability growth rate Γ on the wave number q . The calculations were performed for Skyrme force SIII, temperature $T = 6$ MeV and density $\rho_0 = x \rho_{\text{eq}}$ with $x = 0.3$ and $\rho_{\text{eq}} = 0.1453 \text{ fm}^{-3}$. The solid curve is for the viscous nuclear matter with $\alpha = 9.2$ MeV including both the memory and the Fermi-surface distortion effects. The dashed and dotted lines are the results for the nonviscous liquid: curve (1) is for a nuclear matter neglecting the FSD effect; curve (2) is the result in the presence of the FSD effect and dotted line is the same as curve (1) neglecting the anomalous dispersion term.

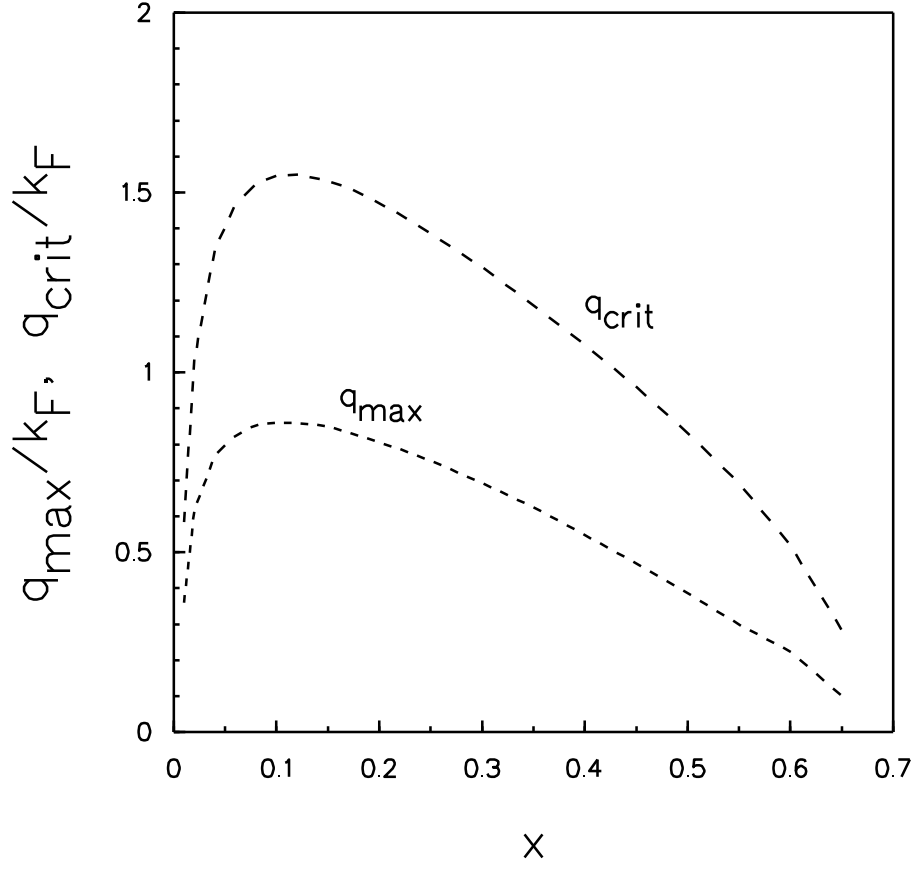


FIG. 4. Dependence of the characteristic wave numbers of the instability growth rate $\Gamma(q)$ on the dimensionless density parameter $x = \rho_0/\rho_{\text{eq}}$. The calculation was performed for Skyrme force SIII and temperature $T = 6 \text{ MeV}$.